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PERSISTENT CURRENT IN TWO SUB-RINGS COUPLED BY ONE EMBEDDED QUANTUM DOT

YA-RU WANG[*], ZHI-MING BAI[†], JI-QING QIU[‡] and YUNG-CHUNG CHEN[§]

[*] *Mathematics and Physics Department, Shijiazhuang Railway Institute, Shijiazhuang 050043, People's Republic of China*
[†] *Physics Department, Hebei University of Science and Technology, Shijiazhuang 050018, People's Republic of China*
[‡] *Mathematics Department, Hebei University of Science and Technology, Shijiazhuang 050018, People's Republic of China*
[§] *Department of Physics, Tunghai University, Taichung, Taiwan*
[†] *baizhiming@tsinghua.org.cn*

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We investigate a two sub-rings mesoscopic system embedded with one quantum dot in common. Owing to the screening cloud of the two sub-rings, the change of the magnetic flux and the number of lattice sites in one sub-ring influence the persistent current not only in itself but also in another. Kondo-assisted (suppressed) tunneling appears in the two sub-rings system constructed by even (odd) number of lattice sites.

Keywords: Persistent current; quantum dot; Aharonov-Bohm ring; variation ansatz method.

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1. Introduction

The investigation in the persistent current (PC) in mesoscopic systems is always given attention in theory and experiments.[1–4] To discuss the quantum phase interference, several interesting setups have been proposed and measured. One setup consists of a quantum dot (QD) embedded in a mesoscopic metal ring.[5–9] When a magnetic field threads in the ring, a persistent current is induced. By the measurement of PC, the coherent transport of electrons through the QD can be understood. Recently, Affleck and Simon[10–12] suggested a setup embedding a QD in one arm of a mesoscopic Aharonov–Bohm (AB) ring to detect the screening cloud in the Kondo regime. When the AB ring circumference is comparable with the fundamental length scale associated with the bulk Kondo temperature, the Kondo resonance is effected strongly.[13] Based on a model that an AB ring is coupled to external leads by a QD, Kicheon and Luis[14] discussed the phase-sensitive transport and showed the Kondo resonance depending on the phase and the total number of electrons (mod 4) in the AB interferometer. They also investigated the persistent current influenced by spin fluctuations in a mesoscopic ring weakly coupled to a QD. It is shown that the Kondo effect gives rise to some unusual features of the persistent current in the limit where the charge transfer between two subsystems is suppressed.[15] Orellana and Pacheco[16] studied the PC and the state density with two quantum rings side-coupled to a quantum wire. They observed that the PC keeps a large amplitude even for the strong ring-wire coupling.

By these motivations, we discuss a QD embedded simultaneously in two mesoscopic circular sub-rings threaded by two different magnetic fluxes. In this geometry, the screening cloud is trapped in the two sub-rings. We are interested in PC in one sub-ring induced by the magnetic flux threading in another sub-ring. We show that since the electron occupation number of the highest energy level in the QD varies with the magnetic flux and the number of lattice sites (NLS), their change in one sub-ring influences PC in another. We find that in the half-filling, if the two sub-rings consist of even NLS, the larger coupling between the QD and sub-ring gives a larger magnitude of PC than $I_0^i[= ev_F^i/(2\pi R_i)$, where v_F^i and R_i are the Fermi velocity and the radius of ith sub-ring], which implies that the Kondo resonance enhances the transmission. In the mean time, PC in another sub-ring is suppressed. On the other hand, if the sub-rings consist of odd NLS, their PCs are always suppressed.

The paper is arranged as follows: In Sec. 2, PC is derived theoretically with the Coulomb interaction in the QD being infinite. Owing to the single electron occupation, a variation ansase method[17] is used to derive expressions of PC in the two sub-rings. In Sec. 3, numerical calculation for PC varying with the NLS is given. The electron occupation number in the QD is plotted in Sec. 4. It is seen that the parity of PC has relation with the occupation number in the QD. We exhibit an asymmetric structure and a peak of PC with the energy level in the QD. The Kondo-assisted persistent current flowing through a QD is shown. Measuring PC in sub-rings provides the coherent information in the experiments. In Sec. 5, we give that the magnetic flux threading in one sub-ring influences PC not only in itself but also in another. Finally, a conclusion is given.

2. Theory

We consider two sub-rings coupled by one QD (or Anderson impurity) in common. Including the QD, the first sub-ring consists of N_1 number of lattice sites and the second one does N_2. The two sub-rings are described by one-dimensional tight-binding model with identical nearest neighbor hoping amplitude t. The coupling coefficients between dot and sub-rings are denoted as t_{Li} and t_{Ri} $(i = 1, 2)$, where $L_i = 1$ and $R_i = N_i - 1$. Only one energy level in QD is assumed relevant. The magnetic flux threading in the first (second) sub-ring is denoted by Φ_1 (Φ_2). A scheme of device is shown in Fig. 1. Hamiltonian of the system can be written as

$$H = H_0 + H_D + H_T \,, \tag{1}$$

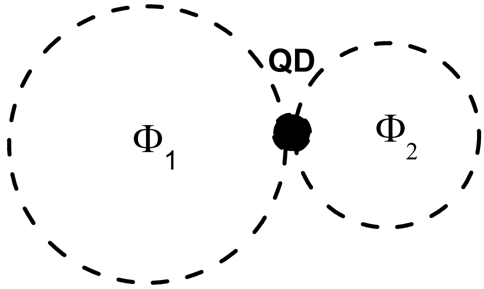

Fig. 1. Schematic diagram of two sub-rings coupled by one quantum dot in common.

where H_0, H_D and H_T represent the noninteracting part for the two sub-rings, the quantum dot and the tunneling term, respectively. These Hamiltonians can be written in the following simplified forms by using a diagonalized basis of H_0:[18]

$$H_0 = \sum_i \sum_{m_i\sigma} \varepsilon_{m_i} c^\dagger_{m_i\sigma} c_{m_i\sigma} ,$$

$$H_D = \sum_\sigma \varepsilon_d d^\dagger_\sigma d_\sigma + U\hat{n}_\uparrow \hat{n}_\downarrow ,$$

$$H_T = \sum_i \sum_{m_i\sigma} (t_{m_i} c^\dagger_{m_i\sigma} d_\sigma + \text{H.c.})$$

where $c^\dagger_{m_i\sigma}$ ($c_{m_i\sigma}$) are the creation (annihilation) operator for electrons in the ith sub-ring ($i = 1, 2$) with spin index σ. ε_d, U, and n_σ representing the single-particle energy, Coulomb potential and the particle number operator in the QD, respectively. The quasi-particle energy of the sub-rings is $\varepsilon_{m_i} = -2t\cos\frac{m_i}{N_i}\pi$ after diagonalizing the Hamiltonian H_0. Renormalized coupling coefficient between the QD and the quasi-particles is

$$t_{m_i} = i\sqrt{\frac{2}{N_i}} \sin\frac{m_i}{N_i}\pi[t_{Li} - (-)^{m_i} t_{Ri} e^{i\phi_i}] ,$$

with $m_i = 1, 2, \ldots, N_i - 1$. The (dimensionless) phase factor is $\phi_i \equiv 2\pi\Phi_i/\Phi_0$, where $\Phi_0 = h/e$ is the flux quantum and Φ_i is the magnetic flux threading in the ith sub-ring. For simplicity, the kinetic energy t is chosen as the energy unit and the coupling between QD and sub-ring is assumed as $t_{Ri} = t_{Li} \equiv t_i$.

For infinite U, one of the simplest ways to investigate the persistent current through the two sub-rings is to adopt a variational ansatz method.[17] Considering the single occupation in the QD and in the leading order of the $1/N_s$ expansion with N_s being the magnetic degeneracy (equal to two in our case), the N_e-particle variational ground state $|\Psi(N_e)\rangle$ is expressed as

$$|\Psi(N_e)\rangle = A\left(1 + \frac{1}{\sqrt{2}} \sum_{i=1,2} \sum_{m_i\sigma}^{\text{occ}} \alpha_{m_i} d^\dagger_\sigma c_{m_i\sigma}\right) |\Omega(N_e)\rangle , \tag{2}$$

where $|\Omega(N_e)\rangle$ denotes the N_e-particle ground state of H_0 for the two isolated sub-rings. The summation in the second term of the right-hand side of Eq. (2) is taken for the occupied level of H_0. Constants A and $\alpha_{m_i\sigma}$ are variational parameters, which are determined by minimization of the function $\langle\Psi(N_e)|H|\Psi(N_e)\rangle - E_0[\langle\Psi(N_e)|\Psi(N_e)\rangle - 1]$. Using the property $d_\sigma|\Omega(N_e)\rangle = 0$ and $c_{m_i\sigma}|\Omega(N_e)\rangle = 0$ ($m_i > N_e/2$ for even number of electrons and $m_i > (N_e+1)/2$ for odd number of electrons in each sub-ring), the ground state energy E_0 of the system can be obtained by the following equation:

$$E_0' = \sum_{i=1,2}\sum_{m_i\sigma}^{\text{occ}} \frac{|t_{m_i}|^2}{E_0' + \varepsilon_{m_i} - \varepsilon_d}\,, \tag{3}$$

where $E_0' = E_0 - E_\Omega$ with E_Ω being the energy of noninteracting ground state $|\Omega(N_e)\rangle$. It can be expressed as $E_\Omega = \sum_{i=1,2}\sum_{m_i\sigma}^{\text{occ}} \varepsilon_{m_i}$, which does not relate to the magnetic flux and the QD. For the half-filling case, the Fermi energy (the electron highest occupation energy level) is zero. We will consider the half-filling in the two sub-rings.

At zero temperature, the persistent current in ith sub-ring is given by $I_i = -\frac{e}{\hbar}\frac{\partial E_0}{\partial \phi_i}$. Using Eq. (3), the persistent current in the ith sub-ring is

$$I_i = \frac{e}{\hbar} D^2 \sum_{i=1,2}\sum_{m_i\sigma}^{\text{occ}} \frac{u_{m_i}}{E_0' + \varepsilon_{m_i} - \varepsilon_d} \sin\phi_i\,, \tag{4}$$

with the renormalization constant A^2

$$D^2 = \left[1 + \sum_{i=1,2}\sum_{m_i\sigma}^{\text{occ}} \frac{|t_{m_i}|^2}{(E_0' + \varepsilon_{m_i} - \varepsilon_d)^2}\right]^{-1}$$

and the function

$$u_{m_i} = -(-)^{m_i}\frac{4}{N_i}\sin^2\left(\frac{m_i}{N_i}\pi\right) t_{Li}t_{Ri}\,.$$

If only one of sub-rings couples with the QD, Eq. (4) coincides with Eq. (7) in Ref. 18. The electron occupation number of the highest occupied energy level of QD is derived as

$$n_d = \sum_{i=1,2}\sum_{m_i\sigma}^{\text{occ}} \frac{|t_{m_i}|^2}{(E_0' + \varepsilon_{m_i} - \varepsilon_d)^2}\,. \tag{5}$$

Once E_0' is known, the persistent current and the electron occupation number of the highest energy level in the QD can be obtained. Since the cross-parity of the NLS only changes the direction of PC induced by the magnetic flux, we only consider that the number of the lattice sites in the two sub-rings has the same even or odd parity simultaneously.

3. Numerical Calculation for PC Varying with the NLS

Since the quasi-particle energy ε_{mi} and the coupling between QD and quasi-particle have relation with the NLS, the ground state energy E_0 is a function of NLS, which makes PC in sub-rings change with the NLS. Figures 2 and 3 give the numerical relation between PC and the NLS for two typically magnetic fluxes.

When the magnetic fluxes threading the two sub-rings are identical, $\phi_1 = \phi_2 = \pi/2$, Fig. 2 shows that the varying of NLS in one sub-ring influences the persistent current not only in itself, but also in another. The parity of the number of lattice sites N_2 determines PC direction and amplitude in sub-ring "2" (mod 4 with the NLS N_2). But this parity has little influence on PC in sub-ring "1" for this special magnetic flux, which is mainly determined by the number of lattice sites in itself (mod 4 with N_1 and mod 2 with N_2). It shows that when the number of lattice sites in sub-ring "1" is an even number, with the increase of the NLS in sub-ring "2", PC in sub-ring "1" increases also, where we are not concerned with the PC direction; but when NLS in sub-ring "1" is an odd number, with the increase of the NLS in sub-ring "2", PC decreases. For large values of N_2, PC in this sub-ring decreases to zero and the current in sub-ring "1" tends to a stable value, which is the same as a ring coupling to a large electronic reservoir by a QD.

Generally, due to the screening electronic cloud being trapped in the two sub-rings, the change of NLS in one sub-ring influences PC not only in itself but also

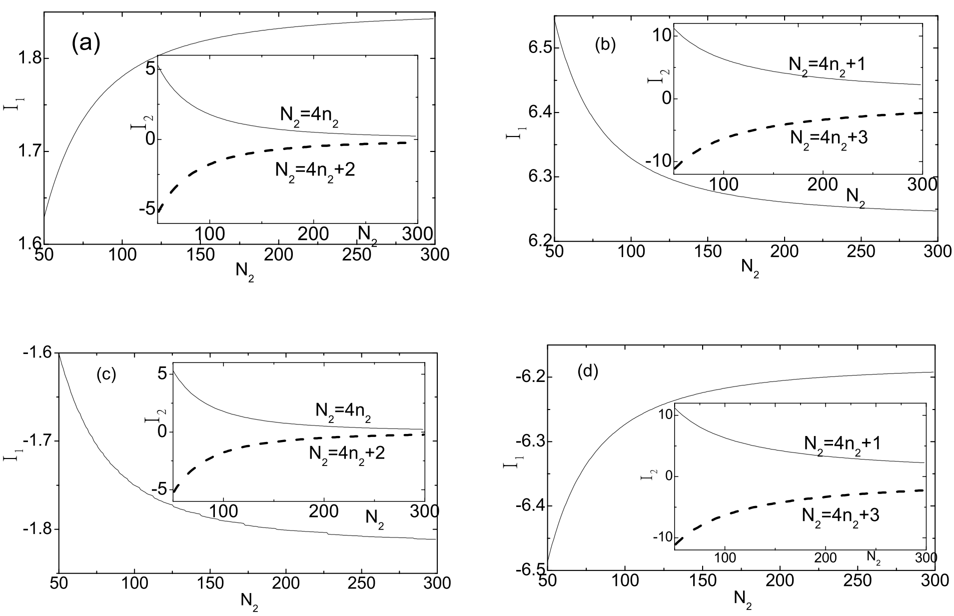

Fig. 2. Persistent current in sub-ring "1" versus to the number of lattice sites N_2 in sub-ring "2" for $e_d = -0.75$, $t_1 = t_2 = 0.3$, $\phi_1 = \phi_2 = \pi/2$. (a) for $N_1 = 4n_1$; (b) for $N_1 = 4n_1 + 1$; (c) for $N_1 = 4n_1 + 2$; (d) for $N_1 = 4n_1 + 3$. The inset shows the current in the sub-ring "2". The unit of current is chosen as $\frac{e}{\hbar}10^{-3}$.

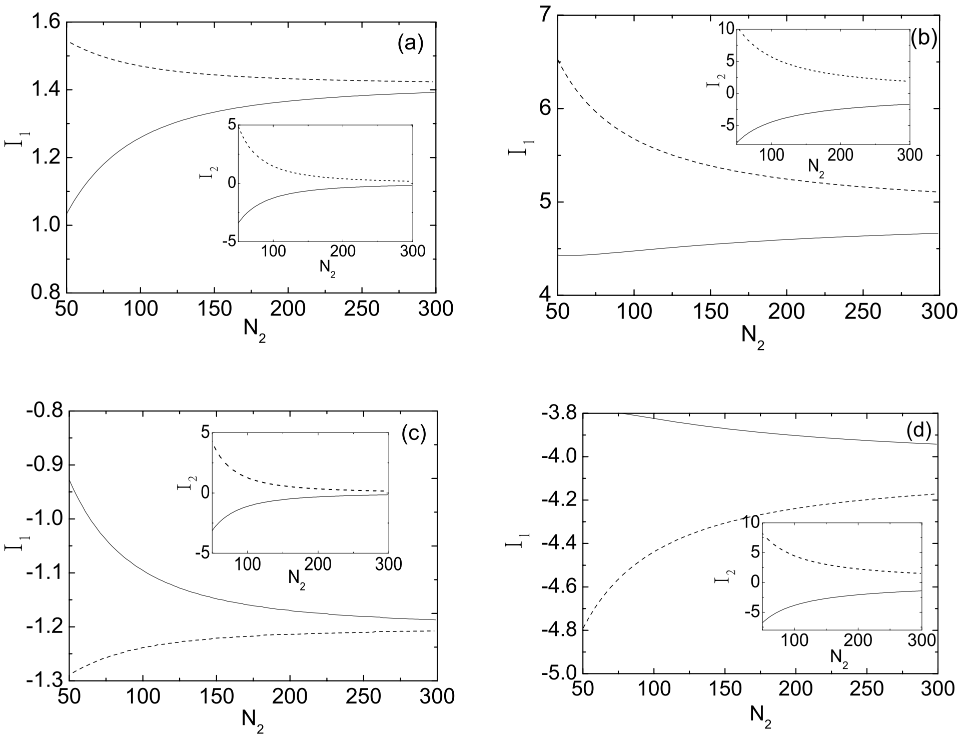

Fig. 3. Persistent current in sub-ring "1" versus NLS N_2 for $e_d = -0.75$, $t_1 = t_2 = 0.3$, $\phi_1 = \phi_2 = \pi/4$. In (a), (c) and their insets, the solid line is for $N_2 = 4n_2 + 2$ and the dashed line is for $N_2 = 4n_2$. In (b), (d) and their insets, the solid line is for $N_2 = 4n_2 + 3$ and the dashed line is for $N_2 = 4n_2 + 1$; (a) for $N_1 = 4n_1$; (b) for $N_1 = 4n_1 + 1$; (c) for $N_1 = 4n_1 + 2$; (d) for $N_1 = 4n_1 + 3$. The inset shows the current in the sub-ring "2". The unit of current is chosen as $\frac{e}{\hbar}10^{-3}$.

in another. Figure 3 shows that when the magnetic flux threading in the two sub-rings is $\phi_1 = \phi_2 = \pi/4$, PCs vary with the increase of the NLS in sub-ring "2". Two features are noted. The first one is that PC in one sub-ring depends strongly on the number of its lattice sites (mod 4 with N_1 and N_2). With the increase of N_2, the property mod 4 with N_2 in I_1 tends to mod 2. Second, if the NSL in sub-ring "2" satisfies $N_2 = 4n_2 + 2$, or $N_2 = 4n_2 + 3$, PC in sub-ring "1" is smaller than that in which the NSL is $N_2 = 4n_2$, or $N_2 = 4n_2 + 1$, where we are still not concerned with the PC direction.

4. PC and Electrons Occupation Number Varying with the Tunable QD Energy Level

When the magnetic fluxes threading the two sub-rings satisfy $\phi_1 = \phi_2 = \pi/2$, respectively, the property of PC for different occupations of the highest energy level of QD by tuning the dot energy level ε_d is investigated in Fig. 4 with different parities. The magnitude of the PC in a uniform ring is $I_0^i = \frac{e}{\hbar}\delta^i$, where

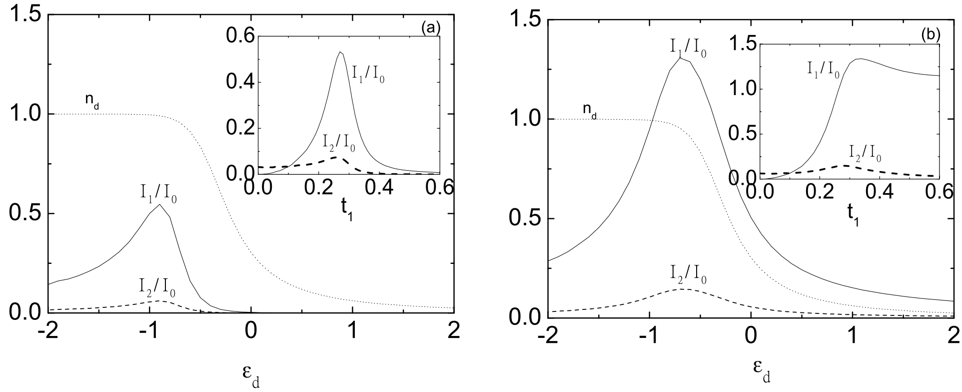

Fig. 4. Persistent current and occupation number versus the energy level of the quantum dot for $t_1 = 0.3$, $t_2 = 0.1$, $\phi_1 = \phi_2 = \pi/2$. The solid line corresponds to the current I_1/I_0, the dashed line does to the current I_2/I_0, and the dot-line does to the occupation number. (a) Kondo-suppressed tunneling for the configuration $(4n_1, 4n_2)$; (b) Kondo-assisted tunneling for the configuration $(4n_1 + 1, 4n_2 + 1)$. Insets are the persistent current versus the coupling t_1, with $\varepsilon_d = 0.75$ and the other coefficients as above.

$\delta^i = 2\pi t/N_i$ is the level spacing of electrons in the ith sub-ring and the NLS N_i is a larger number. The smallest variation of even or odd NLS gives little change for PC except for its direction. In Fig. 4, the I_0^i are all denoted as I_0 for different NLS configurations. We see that PC is mod 4 with the NLS for the general phase ϕ_1 and ϕ_2 contributed by the magnetic flux. Considering the characters in Fig. 2, we plot only two configurations $(N_1, N_2) = (2n_1, 2n_2)$ and $(2n_1 + 1, 2n_2 + 1)$ to describe PC with the tunable energy level at $\phi = \pi/2$. PC exhibits an asymmetric structure and a peak of PC appears. Figure 4(a) displays that the current peaks are smaller than I_0, which expresses Kondo-suppressed tunneling for configuration $(2n_1, 2n_2)$. It happens before the charge in the QD goes into the charge fluctuation area and decreases abruptly after it reaches the peak. Contrarily, Fig. 4(b) shows that electrons in sub-ring "1" are Kondo-assisted tunnelling in configuration $(2n_1 + 1, 2n_2 + 1)$, which happens in the charge fluctuation area. Comparing with the results of Ref. 18, PC is smaller than that in the isolated ring due to another sub-ring's coupling.

When the coupling t_1 is smaller than t_2, insets in Fig. 4 show that I_1 is smaller than I_2. At $t_1 = t_2 (= 0.1)$, the system is symmetrical and PCs in the two sub-rings are identical. With the further increase of the coupling t_1, PC in sub-ring "1" becomes larger than I_2 and Kondo-assisted(suppressed) peak is formed in PCs for the system constructed by even (odd) NLS, respectively. Kondo-assisted(suppressed) tunneling has a tight relation with the coupling between the QD and the sub-ring. After PC reaches the maximum current, further increasing t_1 makes PC decrease. In conclusion, the larger coupling between one sub-ring and QD gives larger PC in itself than in another sub-ring.

5. Magnetic Flux Threading One Sub-Ring Influencing PC in Another

The most convenient way to detect the coherent effect in experiments is to measure PC. Such a measurement has been performed recently on micron sizes ring.

In Fig. 5 we show that when $\phi_1 = \pi/2$, PCs oscillate with the magnetic flux ϕ_2. A striking feature can be seen that the magnetic flux threading one sub-ring not only influences its PC but also does that in the other sub-rings due to the fact that the magnetic flux changes the ground state energy of the system and the electron occupation number of the highest energy level in the QD. Although PCs in the two sub-rings have relation with their NLS, PC profile in the ith sub-ring is mainly determined by NLS in itself (mod 4 with N_i) and has relations approximately $I_i^{(4n_i)}(\phi_{\bar{i}}) \simeq -I_i^{(4n_i+2)}(\phi_{\bar{i}})$, $I_i^{(4n_i+1)}(\phi_{\bar{i}}) \simeq -I_i^{(4n_i+3)}(\phi_{\bar{i}})$, and $I_i^{(4n_i)}(\phi_i) \simeq I_i^{(4n_i+2)}(\phi_i + \pi)$, $I_i^{(4n_i+1)}(\phi_i) \simeq I_i^{(4n_i+3)}(\phi_i + \pi)$ where $i(\bar{i}) = 1(2), 2(1)$. The direction of PC determined by the parity of the lattice sites gives the diamagnetic or

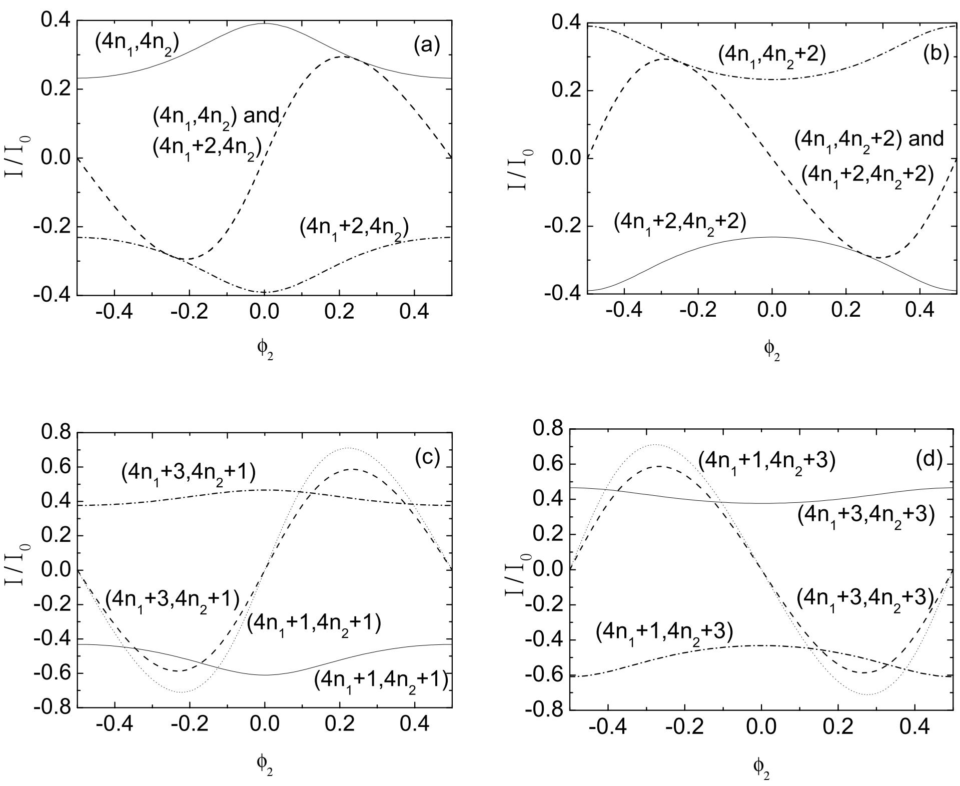

Fig. 5. Persistent currents in two sub-rings vibrates with the phase ϕ_2 for $\varepsilon_d = -0.75$, $\phi_1 = \pi/2$, and $t_1 = t_2 = 0.3$. The solid line and dot-dashed respond to PC in sub-ring "1"; and the dashed line and dot line does to PC in ring "2". (a) For the configurations $(4n_1, 4n_2)$ and $(4n_1+2, 4n_2)$; (b) for the configurations $(4n_1, 4n_2+2)$ and $(4n_1+2, 4n_2+2)$; (c) for the configurations $(4n_1+1, 4n_2+1)$ and $(4n_1+3, 4n_2+1)$; (d) for the configurations $(4n_1+1, 4n_2+3)$ and $(4n_1+3, 4n_2+3)$.

paramagnetic property. If the system is constructed by configurations $(4n_1, 4n_2+2)$, $(4n_1+1, 4n_2+3)$, $(4n_1+2, 4n_2+2)$ or $(4n_1+3, 4n_2+3)$, the magnetic flux threading in the sub-ring "2" enhances PC periodically in the sub-ring "1". Inversely, in the other configuration of the NLS, it decreases the PC periodically. Figure 5(d) and inset in Fig. 4(b) show that if the physical parameters are chosen suitably, such as $t_1 = 0.3$, $t_2 = 0.1$, and $\varepsilon_d = 0.7$, PC in sub-ring "1" reaches a maximum current range due to the Kondo-assisted tunneling, and the magnetic flux threading in sub-ring "2" enhances PC periodically further. Moreover, PC for an even NLS appears to be much larger than that for the odd NLS. Figures 5(a) and 4(b) show that the change of NLS in sub-ring "1" has no influence on PC in sub-ring "2" for the odd NLS, but for the even NLS, Fig. 5(c) and 4(d) show that this influence becomes stronger for $\phi_1 = \pi/2$.

6. Conclusions

In conclusion, we have investigated the persistent current flowing in two mesoscopic Aharonov–Bohm sub-rings coupled by one embedded QD. It is shown that the magnetic flux and the number of lattice sites in one sub-ring influences the persistent current not only in itself but also in another, because the screen cloud is owned by the two sub-rings. It is helpful to know the persistent current in one sub-ring by detecting PC in another. Since the even and odd parity with NLS in the two sub-rings mainly determines the PC direction, the configurations constructed by two adjacent NLS numbers of N_1 and N_2 with different parities give the magnitude of persistent current to be approximately identical, except for its direction.

Acknowledgments

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